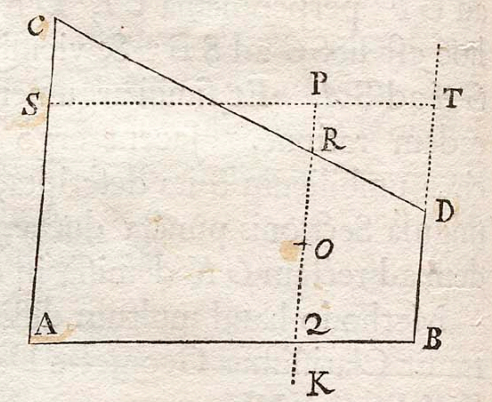
SECT. V.

Inventio Orbium ubi umbilicus neuter datur.

Lemma XVII.

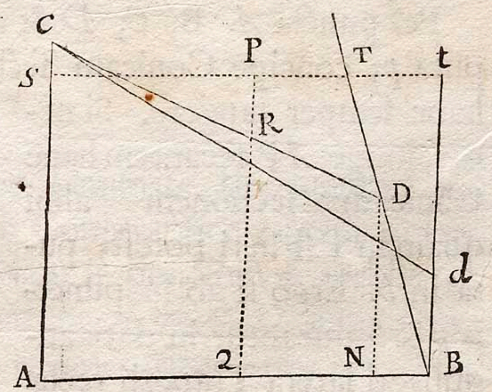
Si a datæ conicæ sectionis puncto quovis P, ad Trapezii alicujus ABCD, in Conica illa sectione inscripti, latera quatuor infinite producta AB, CD, AC, DB, totidem rectæ PQ, PR, PS, PT in datis angulis ducantur, singulæ ad singula: rectangulum ductarum ad opposita duo latera PQ x PR, erit ad rectangulum ductarum ad alia duo latera opposita PS x PT in data ratione.



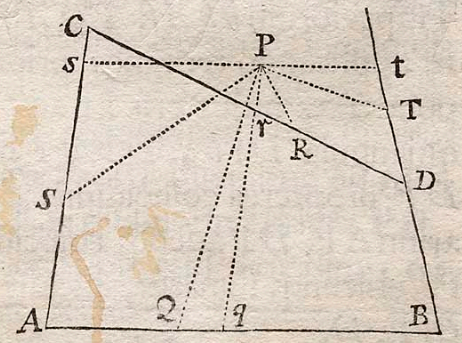
Cas. 1. Ponamus imprimis lineas ad opposita latera ductas parallelas esse alterutri reliquorum laterum, puta PQ & PR lateri AC, & PS ac PT lateri AB. Sintq; insuper latera duo ex oppositis, puta AC & BD, parallela. Et recta quæ bisecat parallela illa latera erit una ex diametris Conicæ sectionis, & bisecabit etiam RQ. Sit O punctum in quo RQ bisecatur, & erit PO ordinatim applicata ad diametrum illam. Produc PO ad K ut sit OK æqualis PO, & erit OK ordinatim applicata ad contrarias partes diametri. Cum igitur puncta A, B, P & K sint ad Conicam sectionem, & PR secet AB in dato angulo, erit (per Prop. 17 & 18 Lib. III *Apollonii*) rectangulum PQK ad rectangulum AQB in data ratione. Sed QK & PR æquales sunt, utpote æqualium OK, OP, & OQ, OR differentiæ, & inde etiam rect-

rectangula PQK & PQ x PR æqualia sunt; atq; adeo rectangulum PQ x PR est ad rectangulum AQB, hoc est ad rectangulum PS x PT in data ratione. Q. E. D.

Cas. 2. Ponamus jam Trapezii latera opposita AC & BD non esse parallela. Age Bd parallelam AC & occurrentem tum rectæ ST in t, tum Conicæ sectioni in d. Junge Cd secantem PQ in r, & ipsi PQ parallelam age DM secantem Cd in M & AB in N. Jam ob similia triangula BTt, DBN, est Bt seu PQ ad Tt ut DN ad NB. Sic & Rr est ad AQ seu PS ut DM ad AN. Ergo ducendo antecedentes in antecedentes & consequentes in consequentes, ut rectangulum PQ in Rr est ad rectangulum Tt in PS, ita rectangulum NDM est ad rectangulum ANB, & (per Cas. 1) ita rectangulum QPr est ad rectangulum SPt, ac divisim ita rectangulum QPR est ad rectangulum PS x PT. Q. E. D.



Cas. 3. Ponamus deniq; lineas quatuor PQ, PR, PS, PT non esse parallelas lateribus AC, AB, sed ad ea utcunq; inclinatas. Earum vice age Pq, Pr parallelas ipsi AC; & Ps, Pt parallelas ipsi AB; & propter datos angulos triangulorum PQq, PRr, PSs, PTt, dabuntur rationes PQ ad Pq, PR ad Pr, PS ad Ps & PT ad Pt, atq; adeo rationes compositæ PQ in PR ad Pq in Pr, & PS in PT ad Ps in Pt. Sed, per superius demonstrata, ratio Pq in Pr ad Ps in Pt data est: Ergo & ratio PQ in PR ad PS in PT. Q. E. D.



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